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## Stochastic Growth and Monetary Policy: the impacts on the term structure of interest rates

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# Stochastic Growth and Monetary Policy: the impacts on the term structure of interest rates\*

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## Abstract

This paper builds a simple, empirically-verifiable rational expectations model for term structure of nominal interest rates analysis. It

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solves an stochastic growth model with investment costs and sticky inflation, susceptible to the intervention of the monetary authority following a policy rule. The model predicts several patterns of the term structure which are in accordance to observed empirical facts: (i) pro-cyclical pattern of the level of nominal interest rates; (ii) countercyclical pattern of the term spread; (iii) pro-cyclical pattern of the curvature of the yield curve; (iv) lower predictability of the slope of the middle of the term structure; and (v) negative correlation of changes in real rates and expected inflation at short horizons.

JEL classification: E32; E43; E52

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## 1 Introduction

This paper provides an answer to two apparently unrelated questions:

- How can an intertemporal equilibrium model adequately fit an arbitrary exogenous term structure of interest rates?
- What is the role of monetary policy in determining the term structure of interest rates?

On the first issue, intertemporal general equilibrium modelling of interest rates still leaves many questions unanswered. As an example, scalar time-homogenous affine equilibrium models<sup>1</sup>, famous for their terse description of an equilibrium economy, which provides tractable and rich analytic results, because of their constant level of reversion, are intrinsically incapable of fitting an arbitrary exogenous term structure. Worse, when tested against more general scalar specifications, they are usually rejected, suggesting either the existence of nonlinearity or of omitted variables (Chan et al. [8] or Aït-Sahalia [1]).

On the second issue, despite the belief that changes in the monetary policy impact on asset returns in general<sup>2</sup> and are a major source of changes in the shape of the yield curve<sup>3</sup>, micro-financial models have not accomplished to properly incorporate it yet. The neglect to deal with macro links leaves unexplained, or even contradicts, certain stylized facts like the pro-cyclical nominal interest rate levels, the countercyclical term spread (Fama & French [13]), or the negative short-run correlation between expected inflation and

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<sup>1</sup>The univariate version of Cox, Ingersoll and Ross [10] can be seen as the most important member of the class.

<sup>2</sup>For example, Thorbecke [28] and Patelis [22] document the existence of a monetary risk premium and show the role of monetary policy in the predictability of the asset returns.

<sup>3</sup>See Mankiw & Miron [17].

the expected future real interest rate in the U.K. (Barr and Campbell [3]).

As macro links, omitted variables and constant reversion levels seem to be the weak points of the scalar time-homogeneous equilibrium models, an attempt is made here to incorporate a macro monetary policy variable into an intertemporal equilibrium model. The goal is to get a simple, empirically-verified rational expectations model for the term structure of nominal interest rates. A model which allows great flexibility in the changes of the yield curve, in response to changes in the macroeconomic environment.

We portray the character of fluctuations in the term structure of nominal interest rates, inflation and aggregate output with staggered price contracts and investment costs, subject to technology shocks and expectational errors by price bargainers. We end up solving a stochastic growth model, subject to investment costs and sticky inflation similar to Fuhrer [15], but susceptible to the intervention of an external authority. The intertemporal optimization implies a complete description of the multi-period expected returns, and the model allows the derivation of a nominal term structure which incorporates the effects of monetary policy. Through discontinuous changes of the short-term nominal interest rate, the Central Bank forces the left-end of the term structure to match an exogenously specified level. This implies a non-zero

net supply of nominal riskless bonds and adds the possibility of jumps in all forward-looking variables. Given that the monetary authority is constrained to keep inflation close to zero, future changes in the controlled rate can be forecasted by looking at the dynamics of the expected inflation and may be incorporated into the shape of the term structure.

The resulting model extends Balduzzi, Bertola & Foresi's [2], Rudebusch's [25], McCallum's [18] and Piazzessi's [23] analyses of the monetary policy impacts on the term structure in the sense that, in an intertemporal equilibrium framework, it allows the joint explanation of more stylized facts. Indeed, with a relatively simple model it is shown that the monetary policy has real effects. We eventually explain: (i) the pro-cyclical pattern of the level of nominal interest rates; (ii) the countercyclical pattern of the term spread<sup>4</sup> (as well as the low sensitivity of long yields to monetary policy changes); (iii) the pro-cyclical pattern of the curvature of the term structure; (iv) the lower predictability of the slope of the middle of the yield curve; and (v) the negative correlation of changes in real rates and expected inflation at short horizons. Though empirical evidence on these facts is abundant in the literature (see for example, Campbell, Lo & MacKinlay [7], Fama & French

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<sup>4</sup>The term spread is defined as the difference between the yield-to-maturities of a long and a short term bond.

[13], Rudebusch [25] and Barr and Campbell [3]) no simple model exists taking simultaneously into account all them. Moreover, implications of the here developed model can be explored in a bond pricing context.

The paper has the following structure. Section 2 presents the empirical patterns, while section 3 reviews the term structure pattern implied by the plain Real Business Cycle model and points out its nominal indeterminacy. Both act as a motivation to section 4, where the proposed model is explained in a representative agent framework. Examples and simulations are performed in section 5, and section 6 concludes. The equivalence between the representative agent and the competitive formulation of the model is fully shown in Appendix 1; Appendix 2 explains the numerical method used in the simulations.

## 2 Some Stylized Facts

This section presents empirical evidences on the movements of the term structure of nominal interest rates, inflation and output, to which the numerical predictions of the theoretical models will be subsequently compared. The empirical pattern of the term structure is reproduced below using the interest



rate data available at the FED of Saint Louis' web site ([www.stls.frb.org/fred](http://www.stls.frb.org/fred)), which are taken from the H.15 Release by the Board of Governors. The seven rates chosen were: 3-Month Treasury Bill Rates (TB3m), 6-Month Treasury Bill Rates (TB6m), 1-Year Treasury Bill Rates (TB1), 3-Year Treasury Constant Maturity Rate (CM3), 5-Year Treasury Constant Maturity Rate (CM5), 7-Year Treasury Constant Maturity Rate (CM7), 10-Year Treasury Constant Maturity Rate (CM10). T-Bills are secondary market rates on Treasury securities and the CM rates are constant maturity yields.<sup>5</sup>

For  $B_t^j$ , the nominal price at  $t$  of the pure discount  $j$ -period bond (or the zero coupon bond that matures in  $j$  periods from  $t$ ), the yield-to-maturity,  $y_t^j$ , is the per period interest rate accrued during the  $j$  periods:

$$B_t^j = (1 + y_t^j)^{-j};$$

what means the yield-to-maturity is the average return on the bond held until maturity.

Because  $B_t^j$  is known at time  $t$ ,  $y_t^j$  is the  $j$ -period riskless nominal rate

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<sup>5</sup>The results to be presented below hold for the Fama & Bliss data set as well, that uses only fully taxable, non-callable bond. The monthly data contain one to five years-to-maturity bonds and cover the period from July 1952 to January 1998, providing 547 observations. The Fama and Bliss data set was constructed by Fama and Bliss [12] and was subsequently updated by the Center for Research in Security Prices (CRSP). The results can be made available upon request.

prevailing at time  $t$  for repayment at  $t + j$ . The *one – period* riskless nominal rate prevailing at time  $t$  for repayment at  $t + 1$  deserves special notation,  $i_t$ :

$$B_t^1 = (1 + i_t)^{-1},$$

and is denoted the spot interest rate. For  $j > l$ , the  $l$  – *period* nominal holding return of the  $j$  – *period* bond between  $t$  and  $t + l$ ,  $h_{t+l, t}^j$ , is the per period interest rate accrued during the  $l$  periods:

$$\frac{B_{t+l}^{j-l}}{B_t^j} = (1 + h_{t+l, t}^j)^l.$$

Given the consumer price index at  $t$ ,  $P_t$ , and the inflation between  $t$  and  $t + l$ ,  $\pi_{t+l, t} = \frac{P_{t+l}}{P_t} - 1$ , the  $l$  – *period* real holding return of the  $j$  – *period* nominal bond,  $r_{t+l, t}^j$ , can be analogously defined as:

$$(1 + r_{t+l, t}^j)^l = \frac{B_{t+l}^{j-l}}{B_t^j} \frac{P_t}{P_{t+l}} = \frac{(1 + h_{t+l, t}^j)^l}{1 + \pi_{t+l, t}}.$$

Note that both  $r_{t+l, t}^j$ ,  $\pi_{t+l, t}$  and  $h_{t+l, t}^j$  only become known at  $t + l$ .

The published data are bond-equivalent yields ( $r_{BEY}$ ) or discount rates ( $r_D$ ). They were transformed to yield-to-maturity by respectively:  $y^j =$

$(1 + r_{BEY} * \frac{j}{100})^{\frac{1}{j}} - 1$  and  $y^j = (1 - r_D * \frac{j}{100})^{-\frac{1}{j}} - 1$ , where  $j$  is time-to-maturity in years. All yields below will be expressed in annualized form.

## 2.1 Pro-cyclical nominal interest rate levels and countercyclical term spread

The evolution of the yields-to-maturity of the three-month and of the ten-year bonds are plotted in Figure 1 with shades added to mark the business cycles. Every white period points one expansion cycle from trough to peak, as classified by the NBER. The gray periods mark the contraction periods from peak to trough. The (i) pro-cyclical pattern of the level of interest rates is clear: the level increases during expansion and decreases during contraction. This may be related to the pro-cyclical pattern of the inflation level, as shown in Figure 2.

Figure 3 shows the evolution of the slope and the curvature of the yield curve <sup>6</sup>. The (ii) term spread presents a countercyclical pattern: the slope of the yield curve is big at the trough and decreases during the cycle to become small at the peak. (iii) Curvature seems to decrease along contractions

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<sup>6</sup>The slope of the yield curve is nothing more than the term spread ( $CM10 - TB3m$ ). The curvature is defined as  $(CM10 - 2 \cdot CM5 + TB3m)$ .

(shades) and to increase during expansions.

From (i), (ii) and (iii), it results that the mean term structure at the trough is a positive sloped, relatively steeper, concave curve, while the mean term structure at the peak is a negative sloped, relatively flatter, convex curve.

## 2.2 Lower predictability of slope of the medium term rates

In the analysis of the term structure, the many versions of the Expectation Theory of the term structure of interest rates have played an important role. Loosely stating, the Expectation Hypothesis says that the expected excess returns on long-term bonds over short term bonds (the term premiums) are constant over time. This means the term premium can depend on the maturity of the bonds but not on time:  $E_t [h_{t+l, t}^j - h_{t+l, t}^k] = f(j, k, l)$ , with  $\frac{\partial f}{\partial t} = 0 \forall j > k > l$ <sup>7</sup>. In its Pure version (the Pure Expectation Hypothesis, PEH), it imposes the term premium to be zero.

If any version of the Expectation Hypothesis holds, the slope of the yield curve is able to forecast interest rate moves, and this predictability is uni-

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<sup>7</sup>The Expectation Hypothesis can be stated in real or in nominal terms.

form along all maturities. For example, to test such a predictability for the “one-period return”, the PEH reduces to check whether the slope,  $b$ , of the regression:

$$(y_{t+1}^{l-1} - y_t^l) = a + b \left( \frac{y_t^l - y_t^1}{l - 1} \right) + e_t, \quad (1)$$

is significant. Indeed, the above hypothesis implies that  $b = 1$  for every  $l$ .

Using monthly zero-coupon bond yields over the period 1952:1 to 1991:2<sup>8</sup>, Campbell, Lo & MacKinlay [7] estimated equations similar to (1) for 2 to 120 months and got the results shown in Table 1.

Besides the  $b$ 's being statistically different from 1, the stylized fact that their results bring to scene is the U-shaped pattern of these slope coefficients: the forecasting power diminishes from the one month to the one year case and then increases up to the ten years case. This means that (iv) the predictability of the middle of the yield curve is lower than those of the edges.

## 2.3 Principal component analysis

Are the previous four stylized facts the result of some identifiable factors? In this regard, principal component analysis might point at least how many factors are relevant for empirical term structure motion. Table 2 shows factors

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<sup>8</sup>Campbell, Loo & MacKinlay [7] use the data from McCulloch and Know [19].

with a pattern similar to the one uncovered by Litterman & Scheinkman's [16].

The first factor has the same sign in all bonds but, different from Litterman & Scheinkman, its impact is higher on the shorter ones. This gives a different interpretation, that the first factor causes moves in the levels and in part of the slope changes. The second factor changes sign from the short end to the long end of the maturities, which means it causes the changes in slope. Finally, the third factor, which has more impact at the short and long ends of the term structure, is interpreted as the curvature factor.

Table 3 shows the proportion of total variance explained by the three factors.

In the FRED data, the first two factors explain most of the movements and almost nothing is left to factors 3 and further <sup>9</sup>.

Using the FRED 1969-2000 sample and varying frequency, we have performed other principal component analyses (not shown) and obtained that, once frequency is increased, the first factor loses explanatory power to the

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<sup>9</sup>Litterman & Scheinkman [16] used weekly observations, from January 1984 to June 1988, of maturities 6-month, 1, 2, 5, 8, 10, 14 and 18-year. They got averages of 89.5 %, 8.5 % and 2 % for the proportion of the total explained variance by the first three factors. Their different result might have been caused by the different frequency and length of the time series, or span of the maturities.

second and third ones. This is a weak evidence that the 2nd. and 3rd. factors are more important in explaining short run movements <sup>10</sup>.

## 2.4 Negative correlation of changes in real rates and expected inflation at short horizons

Well known in the fixed income theory is the Fisher hypothesis that there is no correlation between the expected inflation and the real interest rates: nominal interest rates change to fully compensate for expected inflation variations.

However, this hypothesis is not verified once taken to data: (iv) there may exist negative correlation between expected inflation and real interest rate at short horizons. This fact is shown for example by Barr and Campbell [3], who, working with U.K. data, find correlations of changes in real rates and expected inflation of -0.69, -0.06 and -0.08 for 1-year, 5-year and 10-year, respectively. The significant negative correlation got at a short horizon is puzzling, since it is expected that investors increase (decrease) their asked nominal interest rates every time a higher (lower) inflation is expected.

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<sup>10</sup>This is also an evidence that L&S different results might have been caused by the different length of the time series or span of the maturities.

### 3 A Simple Intertemporal Equilibrium Theory of the Term Structure with production

Because intertemporal optimization models imply a complete description of the multi-period expected returns, and the term structure of interest rates is merely the plot of these observed returns, they are suitable as the micro-foundation of a term structure model.

In the Real Business Cycle (RBC) model with labor supplied inelastically, the representative agent maximizes:

$$E_t \left[ \sum_{i=t}^{\infty} \beta^{i-t} u(c_i) \right] \quad (2)$$

with:  $u'(\cdot) \geq 0$ ,  $u''(\cdot) < 0$ ; subject to the budget constraint:

$$\begin{aligned} & c_t + k_{t+1} + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j \\ &= \theta_t k_t^\alpha + (1 - \delta) k_t + \frac{1}{(1 + \pi_{t,t-1})} \left[ (1 + i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^{j+1}} b_t^j \right] - \tau_t; \end{aligned} \quad (3)$$

to the technology shock AR(1) dynamics:

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t, \quad \rho \in (0, 1), \varepsilon_t \sim N(0, \sigma_\varepsilon^2); \quad (4)$$



and the transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^t k_t = 0; \quad (5)$$

$$\lim_{t \rightarrow \infty} \beta^t \sum_{j=1}^{\infty} \frac{B_t^j}{P_t} b_t^j = 0; \quad (6)$$

where:

$c$  stands for real consumption;

$k$  is the real capital stock;

$\theta$  is the productivity shock;

$0 < \alpha < 1$  is the capital elasticity<sup>11</sup>;

$\delta$  is capital depreciation;

$(1 + \pi_{t,t-1}) = \frac{P_t}{P_{t-1}}$  is the inflation between  $t - 1$  and  $t$ , with the price index  $P_t$  not known before  $t$ ;

$(1 + i_t)$  is the nominal interest rate of the *one-period* bond held between  $t - 1$  and  $t$ , known at  $t - 1$ ;

$B_t^j$  is the nominal price of the  $j - period$  bond;

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<sup>11</sup>The production function  $f(k, \theta) = \theta_t k_t^\alpha$  presents the usual conditions:

$$f_1(.) \geq 0, f_2(.) > 0, f_{11}(.) \leq 0, f_1(0,.) = \infty, f_1(\infty,.) = 0;$$

$b_t^j$  is the quantity of the bond the consumer carries from  $t - 1$  to  $t$ , and  $j$  is the number of periods to maturity;

$b_t^0$  is the quantity of the bond redeemed at  $t$ ;

and  $\tau_t$  are real taxes.

Because labor is inelastically supplied, the production function is presented in terms of per-capita capital, and the above formulation couches the case of a constant return-to-scale production function. Also, to make presentation lighter, instead of the usual normalization of nominal unit price at maturity,  $B_t^0 = 1 \forall t$ , we assume that the next-to-mature bond costs one nominal unit and is worth  $(1 + i_{t+1})$  nominal units at redemption.

From the above, the representative agent value function can be posed as:

$$\begin{aligned}
 & V(k_t; b_t^j; j > 0; \theta_t) \\
 = & \max_{c, k, b} \left\{ \begin{aligned} & u(c_t) + \beta E_t V(k_{t+1}, b_{t+1}^{j>0}, \theta_{t+1}) \\ & - \lambda_t \left[ \begin{aligned} & c_t + \tau_t + k_{t+1} + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j - \theta_t k_t^\alpha - (1 - \delta) k_t \\ & - \frac{1}{(1 + \pi_{t,t-1})} \left[ (1 + i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^j} b_t^j \right] \end{aligned} \right] \end{aligned} \right\}, \tag{7}
 \end{aligned}$$

and solved to result in the agent's optimal allocation rules:

$$u'(c_t) = \beta E_t \left\{ [\alpha \theta_{t+1} k_{t+1}^{\alpha-1} + (1 - \delta)] u'(c_{t+1}) \right\} \quad (8)$$

$$\frac{u'(c_t)}{(1 + i_{t+1})} = \beta E_t \left[ \frac{1}{(1 + \pi_{t+1,t})} u'(c_{t+1}) \right]; \quad (9)$$

and

$$u'(c_t) \frac{B_t^j}{P_t} = \beta E_t \left[ \frac{B_{t+1}^{j-1}}{P_{t+1}} u'(c_{t+1}) \right] \quad \forall j; \quad (10)$$

taking prices as given.

Recursion on (10) and the law of iterated expectations implies the  $l - period$  real holding return of the  $j - period$  nominal bond  $(r_{t+l,t}^j)$ :

$$1 = \beta^l E_t \left[ \frac{B_{t+l}^{j-l}}{B_t^j} \frac{P_t}{P_{t+l}} \frac{u'(c_{t+l})}{u'(c_t)} \right] = \beta^l E_t \left[ (1 + r_{t+l,t}^j)^l \frac{u'(c_{t+l})}{u'(c_t)} \right] \quad \forall j \text{ and } l > 1, \quad (11)$$

and gives the whole real term structure implied by the model.

Inasmuch as the yield-to-maturity of every  $l - period$  bond  $(y_t^l)$  is known for certainty at  $t$ ,  $(1 + y_t^l)^l = \frac{B_{t+l}^0}{B_t^l}$ , it can be taken out of the expectation operator, resulting in:

$$\frac{1}{(1 + y_t^l)^l} = \frac{B_t^l}{B_{t+l}^0} = \beta^l E_t \left[ \frac{1}{1 + \pi_{t+l, t}} \frac{u'(c_{t+l})}{u'(c_t)} \right] \quad \forall l; \quad (12)$$

that provides the whole nominal term structure.

It is trivial that, for  $l = 1$ ,  $y_t^1 = i_{t+1}$ ; and the above formula simplifies to:

$$1 = \beta E_t \left[ (1 + r_{t+1, t}^1) \frac{u'(c_{t+1})}{u'(c_t)} \right] = \beta E_t \left[ \frac{1 + i_{t+1}}{1 + \pi_{t+1, t}} \frac{u'(c_{t+1})}{u'(c_t)} \right]; \quad (13)$$

where the spot rate  $i_{t+1}$  can be put outside the expectation if desired.

From (12), again by use of the law of iterated expectations, we obtain:

$$\begin{aligned} & \frac{1}{(1 + y_t^{2l})^{2l}} \\ = & \frac{1}{(1 + y_t^l)^l} E_t \left[ \frac{1}{(1 + y_{t+l}^l)^l} \right] + Cov_t \left( \frac{\beta^l}{1 + \pi_{t+l, t}} \frac{u'(c_{t+l})}{u'(c_t)}, \frac{1}{(1 + y_{t+l}^l)^l} \right) \quad \forall l; \end{aligned} \quad (14)$$

which is a generalized version of the PEH, adjusted for the risk premium

$$Cov_t \left( \frac{\beta^l}{1 + \pi_{t, t+l}} \frac{u'(c_{t+l}^*)}{u'(c_t^*)}, \frac{1}{(1 + y_{t+l}^l)^l} \right).$$

Equation (14) means the PEH holds only in the special cases where the risk premium is zero.

Also, working on (12), results in the generalized “one-period return” PEH:

$$\begin{aligned} & (1 + y_t^1) \\ = & (1 + y_t^l)^l E_t \left[ \frac{1}{(1 + y_{t+1}^{l-1})^{l-1}} \right] + \frac{Cov_t \left( \frac{1}{1 + \pi_{t+1, t}} \frac{u'(c_{t+1})}{u'(c_t)}, \frac{1}{(1 + y_{t+1}^{l-1})^{l-1}} \right)}{E_t \left[ \frac{1}{1 + \pi_{t+1, t}} \frac{u'(c_{t+1})}{u'(c_t)} \right]} \quad \forall l; \end{aligned} \quad (15)$$

as called by Campbell, Lo and MacKinlay [7]. Again, only when the risk premium is zero, does the “one-period” PEH hold.

The agent’s optimal conditions allow us to define:

$$M_{lt} = \beta^l \frac{u'(c_{t+l})}{u'(c_t)} \quad (16)$$

as the stochastic discount function (or the pricing kernel); which in the present model is equivalent to the intertemporal marginal rate of substitution in consumption.

### 3.1 Equilibrium without external intervention: inflation and nominal interest rate indeterminacy

An equilibrium sequence is defined as a set of stochastic vectors

$(\theta_t, k_{t+1}, c_t, i_{t+1}, \pi_{t,t-1}, r_{t+l,t}^j, b_{t+1}^j, \tau_t)$  satisfying the f.o.c.'s and the market clearing conditions for every  $t$ .

Without external intervention, the exogenous supply of bonds is zero:

$$b_t^j = 0 \quad \forall j;$$

as well as taxes  $\tau_t = 0$ , and, given (4), the consumers' decision simplifies to split wealth between capital and consumption by obeying (8) and the simplified budget constraint:

$$c_t = \theta_t k_t^\alpha + (1 - \delta) k_t - k_{t+1}, \tag{17}$$

for every  $t$ .

The initial capital stock, the technology dynamics (4), and the transversality condition (5) define the saddle path expected to be followed by  $(k, c)$  in the system (8) and (17). Substitution of (17) into (8) defines a stochastic difference equation in  $k$  that, given the initial capital stock, initial technology and (5), obtains the optimal capital path  $(k^*)$  and provides the inputs to obtain the optimal consumption path  $(c^*)$  by (17). The above hypotheses are enough to guarantee that the distribution of optimum aggregate capital

converges pointwise to a limit distribution when returns are decreasing:  $k$  is pushed to the level  $k_{ss}$  where the expected marginal productivity of capital equals the rate of time preference:  $\alpha k_{ss}^{\alpha-1} - \delta = (1/\beta) - 1$ . When returns-to-scale are constant, they are as well enough to guarantee that the rates of growth converge pointwise to a limit distribution<sup>12</sup>.

The application of  $\{c_t^*\}_{t=0}^\infty$  to (11) endogenously determines the expected  $l$  - *period* real returns on a  $j$  - *period* nominal bonds from  $t$  to  $t + l$ :

$$1 = \beta^l E_t \left[ (1 + r_{t+l,t}^j)^l \frac{u'(c_{t+l}^*)}{u'(c_t^*)} \right] \quad \forall j > l; \quad (18)$$

and gives the whole expected real term structure implied by this equilibrium.

We now define what we understand by neutral values.

**Definition 1** At any time  $t$ , the endogenous variables values are neutral, denoted  $(k_{t+1}^N, c_t^N, i_{t+1}^N, h_{t+1,t}^{N,j}, \pi_{t+1,t}^N, r_{t+1,t}^{N,j}, b_{t+1,t+1}^{N,j})$ , when the real stock of bonds is fully rolled over with no portfolio rebalance:

$$b_{t+1}^{j+1} - b_t^j = 0 \quad \forall j.$$

This means that we qualify all interest rates as neutral when they are

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<sup>12</sup>See Brock [5] for the proof.

obtained without changes in the bonds' maturity profile. There is no net external intervention, in the sense that the debt-credit profile is kept constant. Thus, within a period, the neutral values are nothing more than those for which the private sector's net demand for every maturity bond is zero, what means people do not sell bonds to finance capital or the other way around.

Because there is a stochastic shock in the production function, the neutral real spot rate fluctuates around a trend defined by the optimal capital path. For example: if  $k_t$  is increasing along time and the production function presents decreasing returns-to-scale, the productivity trend is decreasing and real neutral rate is expected to decrease as the economy tends to the steady state.

Without an external intervention, the real interest rates, given by (18), are completely defined by (4), (8), (17) and (5). Equation (13) is nothing more than the Fisher relation that defines next period inflation given the spot nominal interest rate, or the other way around. Because the expected spot real interest rate is completely determined by the real factors and is every time the expected marginal productivity of capital, expected inflation sensitivity to the level of the nominal interest rate is one, what means no correlation between nominal and real variables.



Although the inflation and nominal rate indeterminacies are a consequence of having more variables than equations, the inclusion of a cash-in-advance restriction or a fiscalist-theory type of reasoning does not change the above conclusions. Due to this one-to-one correspondence between  $i$  and  $\pi$ , there is no cyclical pattern (i) in the level of the nominal term structure, (ii) or in that of the term spread, (iii) or in that of the curvature. (iv) The predictability of the slope of the yield curve is good and equally credible for every maturity. Moreover, (v) there is no correlation between expected inflation and the real interest rate since the real interest rates vary with the marginal productivity of capital and the Fisher hypothesis holds.

Summing up, system (4), (8), (17) and (5) alone does not split the changes in the nominal rate into changes in the real rate and inflation, and is not of great use in explaining how monetary policy affects real activity and inflation. Basically, it assumes neutrality (and superneutrality) and thus thwarts the possibility that nominal interest rate and inflation vary independently. Quite unrealistic, inflation reduction to zero can be done in one painless down-move of the nominal rate to the expected marginal productivity level with no impact on the real activity.

Notwithstanding, there exists one degree of freedom in the above model

to couch an ad hoc assumption, and this is done, in conjunction with inflation stickiness, in section 4.

## 4 The Model

The proposed model describes a closed economy<sup>13</sup> with firms and capital accumulation, subject to investment cost and staggered price contracting, and susceptible to the intervention of a monetary authority. For presentation purposes, we develop the main ideas in the representative agent framework.

The equivalence with a more detailed economy, where consumers and firms interact in a world of staggered price contracting, is shown in Appendix 1.

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<sup>13</sup>As pointed in Meltzer (1995) pp.50, in an open economy, the exchange rate would be just one more of the many relative prices in the transmission process, without altering the basic results.

## 4.1 The Real Side with investment costs

The representative agent maximizes (2), subject to a budget constraint slightly different from (3):

$$\begin{aligned} & c_t + \left[ k_{t+1} + \varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right] + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j \\ &= \theta_t k_t^\alpha + (1 - \delta) k_t + \frac{1}{(1 + \pi_{t,t-1})} \left[ (1 + i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^{j+1}} b_t^j \right] - \tau_t, \end{aligned} \quad (19)$$

and to (4), (5), (6); where:  $\varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2$  is the cost of adjustment, and the other variables have the previous stated meaning.

Now, the representative agent value function can be posed as:

$$\begin{aligned} & V(k_t; b_t^j; j > 0; \theta_t) \\ &= \max_{c, k, b} \left\{ \begin{aligned} & u(c_t) + \beta E_t V(k_{t+1}, b_{t+1}^{j>0}, \theta_{t+1}) \\ & - \lambda_t \left[ \begin{aligned} & c_t + \tau_t + \left[ k_{t+1} + \varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right] + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j \\ & - \theta_t k_t^\alpha - (1 - \delta) k_t - \frac{1}{(1 + \pi_{t,t-1})} \left[ (1 + i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^{j+1}} b_t^j \right] \end{aligned} \right] \end{aligned} \right\}; \end{aligned} \quad (20)$$

and the solution is similar to the one in section 3, except that:

$$\begin{aligned}
& \left[ 1 + 2\varphi \left( \frac{k_{t+1}}{k_t} - 1 \right) \frac{1}{k_t} \right] u'(c_t) \\
= & \beta E_t \left\{ \left[ \alpha \theta_{t+1} k_{t+1}^{\alpha-1} + (1 - \delta) - 2\varphi \left( \frac{k_{t+2}}{k_{t+1}} - 1 \right) \frac{k_{t+2}}{k_{t+1}^2} \right] u'(c_{t+1}) \right\},
\end{aligned} \tag{21}$$

replaces (8).

## 4.2 Contracting Specification and the Inflation Dynamics

Once accounted the investment costs, it is assumed that consumption and capital goods ( $c$  and  $k$ ) are the same final good, which is the aggregation of two differentiated goods produced, consumed and invested together in a fixed proportion of half each. Although undesirable, the no substitutability between these (differentiated) component goods simplifies matters and buttresses a staggered price contracting similar to Fuhrer & Moore [14]. In our paper, agents negotiate the nominal price contracts of the two final goods, that remain in effect for two periods. As the model hypothesizes that production, consumption and investment are split between these two goods, the aggregate price index at  $t$  is defined as the geometric mean of the contract

prices:

$$P_t = X_t^{\frac{1}{2}} X_{t-1}^{\frac{1}{2}}; \quad (22)$$

where:

$X_t$  is the contract price

and  $P_t$  is the aggregate price index at  $t$ .

Agents set nominal contract prices so that the current real contract price equals the average real contract price index expected to prevail over the life of the contract, adjusted for excess demand conditions:

$$\frac{X_t}{P_t} = E_t \left[ \frac{X_{t+1}}{P_{t+1}} \right]^{\frac{1}{2}} \frac{X_{t-1}}{P_{t-1}}^{\frac{1}{2}} Y_t^\gamma; \quad (23)$$

where the excess demand term  $Y_t$  was parametrized as  $Y_t = e^{y_t}$ . With this,  $y_t$  is the excess demand which can be calculated from the budget constraint (19) as:

$$\begin{aligned} y_t &= c_t + \left[ k_{t+1} + \varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right] - (\theta_t k_t^\alpha + (1 - \delta) k_t) \\ &= - \left( b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j + \tau_t \right) + \left( \frac{1}{(1 + \pi_{t,t-1})} \left[ (1 + i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^{j+1}} b_t^j \right] \right). \end{aligned} \quad (24)$$

Considering the expression after the first equality signal, the two first members describe total demand for goods, while the last one (the big expression between brackets) is the supply of goods. Thus, excess demand can be read as the private sector's net demand for bonds, and there is no excess demand ( $y_t = 0$ ) when variables from  $t$  to  $t+1$  are neutral (as stated in the Definition).

Equation (23) causes the inflation dynamics:

$$(1 + \pi_{t,t-1}) = (1 + \pi_{t-1,t-2})^{\frac{1}{2}} (1 + E_t [\pi_{t+1,t}])^{\frac{1}{2}} (Y_t Y_{t-1})^\gamma \Omega_t, \quad (25)$$

where  $\Omega_t$  is the expectational error, and allows inflation stickiness in the present model. Note that if expressed in log terms, (25) gives an expression very similar to the one in Fuhrer & Moore [14], which will be used in the simulations in section 5 below.

### 4.3 The Monetary Authority Intervention and the Role Played by Money

Since we are interested on the study of moves in the yield curve, and not on the study of optimal monetary policy rules, we don't care about objective functions of the monetary authority and related issues. It is enough that the

monetary authority be concerned about inflation, have funds to intervene in the bond market, and knows its dynamics is given by (25). This being the case, it is prone to control the one-period spot interest rate to fight inflation. Due to operating constraints, it is assumed, without loss of generality, that it uses the rule:

$$i_{t+1} = i_t + v_t, \tag{26}$$

where:

$$v_t = \left\{ \begin{array}{l} 0, \text{ with probability : } (1 - \varsigma |\pi_{t-1}|) \\ e^{\frac{\pi_{t-1}}{|\pi_{t-1}|}}, \text{ with probability : } \varsigma |\pi_{t-1}| \end{array} \right\};$$

and  $e$  and  $\varsigma$  are positive constants <sup>14</sup>.

In other words, the spot rate tends to remain constant from period to period, except for jumps whose probability is an increasing function of the inflation level. If inflation is positive the eventual jump is positive, and if inflation there is deflation the jump is negative. When inflation grows, the probability of jumps increases and so the expected value of the next

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<sup>14</sup>(26) implies the monetary authority inflation targeting is zero. This assumption can be relaxed by subtracting a constant (or a variable) from  $\pi_{t-1}$ .

period spot rate. Because inflation is persistent, policy only reverts when the inflation target has been mostly reached.

The key to our model is monetary authority behavior in the bond market. It acts buying or selling one-period bonds that pay riskless nominal interest rate  $(1 + i_{t+1})$ , but risky real interest rate:

$$\left( \frac{1 + i_{t+1}}{1 + \pi_{t+1,t}} \right),$$

revealed at  $t + 1$ . Besides, the authority runs no deficit, what forces it to charge the individuals a lump sum tax to payoff the net interest:

$$\tau_t = \left[ \frac{1 + i_t}{1 + \pi_{t,t-1}} - 1 \right] b_t^a \quad \forall t > 0, \quad (27)$$

where  $b_t^a$  stands for the per capita bond demand.

As individuals receive the full proceeds of bonds they hold and are charged lump sum, they choose to long or short the one-period bond once its real expected return diverges from the expected neutral rate. Thus, although lending to or borrowing from the monetary authority are just simple storage in the aggregate, non-zero net demand for one-period government bonds shows up due to the non-cooperative individual behavior induced by the tax system.



Not only the above rule makes it easy to forecast tomorrow's spot rate, but it also answers for the system stability as long as it guarantees that inflation does not explode, providing the long run level of the variables. Stability is the cause for the long rates' low sensitivity to monetary policy changes: given the parameters, long run values are implied, and they are the ones that weight most in the valuation of long term bonds.

No explicit cash motive has been couched; but, without the cash-in-advance restriction, why would society use money and bear the costs of monetary policy? Like Woodford [27], it is assumed that modelling the fine details of the payments system and the sources of money demand is inessential to explain how money prices are determined or to analyze the effects of alternative policies on the inflation path or on other macro variables.

Though buttressing the use of money is not a goal of this paper, we point out a simple fact of life: money allows specialization, what causes productivity gains, and that is why society copes with the monetary authority and its effects. The economic system is enormously more efficient with than without money and the monetary authority. Loosely modelling, at the real side, there exist storable goods and two possible production systems. The monetary system,  $f^M$ , makes use of money, allows specialization and is thus much

more productive than the other,  $f^B$ , non-monetary, non-specialized system:  $f^M(k, \theta) \gg f^B(k, \theta) \quad \forall k$ . Although storage is also possible, it is greatly inefficient: production always generates net goods, even after accounting for all sort of costs and when  $\theta = \theta_{\text{inf}}$ , while storage just returns the amount stored back.

It is just being assumed here that the gain from being a monetary economy is discrete and independent of the inflation level, up to an inflation upper bound above which the economy retraces to the non-monetary system ( $f^B$ ). The dread to bear such a retrace is what justifies the external authority concern about the inflation level. Due to system stability, it will always be assumed that inflation is below the upper bound and  $f = f^M$ .

Since real balance effects do not appear in the inflation dynamics (25), nor the monetary authority controls the money supply <sup>15</sup>, the inflation level determination does not depend upon money demand. The key to analyze the determination of the inflation level without explicit reference to money is to model inflation as a function of the level of the real interest rate, and nominal interest rate as a function of past inflation. This makes real quantities dependent upon the level of inflation and allows the introduction of the

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<sup>15</sup>When the monetary authority controls interest rates, money becomes endogenous.

monetary authority and its policy effects.

#### 4.4 Equilibrium with Intervention Possibility

Equation (19) can be simplified a bit. Because the Central Bank only intervenes in the *one-period* bond market, only  $b_t^0$  and  $b_{t+1}^1$  can be different from zero and the exogenous supply of the bonds longer than one period is zero:  $b_t^j = 0 \ \forall \ j > 1$ . In the representative agent world, equilibrium means:

$$b_t^a = b_t^0;$$

by the intervention policy (27), the economy budget constraint (19) becomes:

$$c_t = \theta_t k_t^\alpha + (1 - \delta) k_t + b_t^0 - \left[ k_{t+1} + \varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right] - b_{t+1}^1 \quad (28)$$

and the excess demand (24):

$$\begin{aligned} y_t &= c_t + \left[ k_{t+1} + \varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right] - (\theta_t k_t^\alpha + (1 - \delta) k_t) \\ &= -b_{t+1}^1 + b_t^0. \end{aligned} \quad (29)$$

with  $c_t$ ,  $k_{t+1}$ ,  $b_{t+1}^1$  optimally given by (21) and (9).

Inflation dynamics simplifies to:

$$(1 + \pi_{t,t-1}) = (1 + \pi_{t-1,t-2})^{\frac{1}{2}} (1 + E_t [\pi_{t+1,t}])^{\frac{1}{2}} \exp \{ \gamma (-b_{t+1}^1 + b_{t-1}^0) \} \Omega_t, \quad (30)$$

The economy equilibrium sequence  $(\theta_t, i_{t+1}, c_t, k_{t+1}, b_{t+1}^1, \pi_{t,t-1}, r_{t,t-1}^j)$  is now given by the system of six simultaneous equations (28), (4), (30), (26), (21) and (13), and the transversality conditions (5) and (6), given the initial values for  $\pi_{0,-1}$ ,  $b_1^0$ ,  $k_1$  and  $i_1$ .

## 4.5 Understanding the model dynamics

The monetary transmission mechanisms are Tobin's Q theory of investment and the wealth effects on consumption: the spot rate change sponsors consumption and portfolio responses with real effects.

Although in the representative agent framework, we are able to argue in terms of the Q-theory of investment. It is possible to get the evolution of marginal Tobin's Q:

$$Q = 1 + 2\varphi \left( \frac{k_{t+1}^*}{k_t^*} - 1 \right) \frac{1}{k_t^*}; \quad (31)$$

for the optimal capital sequence  $\{k_t^*\}_{t=0}^\infty$ .

To illustrate the implications of the model, we can make use of phase diagrams to look at the implied dynamics and the evolution of the term structure along time. Figure 4 shows the saddle path for the pair  $(Q, k)$ .  $Q$  is above unit for increasing  $k$  and is below unit for decreasing  $k$ .

The steady state is the point where the effective output equals the potential one, and there is no excess demand ( $y_t = 0$ ). In this case, at every technology shock that improves (worsens) efficiency,  $\Delta Q = 0$  moves north-east (southwest). The effect is similar in the case of monetary interventions that lower (rise) the real interest rate. However, as these last interest changes are transitory, a backwards move in the  $\Delta Q = 0$  curve is expected to take place sometime in the future.

The variety of term structure shapes and dynamics allowed makes comprehensive illustration unfeasible, but intuition can be gained in the analysis of simple cases. For example, without inflation, the left diagram in Figure 5 shows the dynamics of  $Q$  and  $K$ , and the right diagram shows the implied dynamics of the real term structure. It is the case without intervention of an economy's growth path.

From equation (18) it can be inferred that the real term structure becomes

flatter as the economy comes close to the steady-state ( $t = s.s.$ ), since the ratios of two different time consumptions approach unity (and the real yields approach  $\beta^{-1}$  for every maturity).  $y_t^j$  is given by:

$$y_t^j = \frac{1}{\beta} \left( E_t \left[ \frac{u'(c_{t+j})}{u'(c_t)} \right] \right)^{\frac{-1}{j}} - 1, \quad \forall j,$$

and at  $t = 0$  ( $k_0$  below  $k_{ss}$ ), the real term structure is downward sloping since  $c_t$  is expected to grow at decreasing rates. The just described expansion path contrasts the initial negative slope of the real term structure with the empirical initial positive slope of the nominal term structure shown in section 2. This stress our that plain RBC models, or the univariate version of Cox, Ingersoll and Ross, aren't good enough to explain the nominal term structure. Something practitioners in the financial markets are well aware of.

Figure 6 shows what happens when a temporary increase in the real spot interest rate is expected at a certain date and for a certain period, due to a tight of the Central Bank to fight increasing inflation<sup>16</sup>: once the tight becomes expected,  $Q$  jumps down and  $K$  begins to decrease up to the time when the change happens (at  $T$ ). Between the effective tight and the time

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<sup>16</sup>This is an unrealistic exercise with didactical purposes only. Central Bank's interventions are uncertain as well as their duration.

policy is again loosened,  $Q$  increases, while  $K$  first decreases, to increase after  $Q$  reaches unit.  $(Q, K)$  changes happen so that when policy reverts to loose again (at T'), the pair is over the original saddle and goes to the steady-state.

Figure 7, on the other hand, shows what happens when the time of the target is uncertain. Once the change becomes justifiable by “high” inflation,  $Q$  jumps to an intermediary saddle path, located in accordance with the probability of change. While the change does not happen, inflation is increasing and the intermediary saddle moves southwest (due to the increasing probability), bringing together the pair  $(Q, K)$ . Once the tight takes place (at T),  $Q$  jumps again to a point that depends on the expected future monetary policy.

The combination of the real spot interest rate with the inflation dynamics allows to obtain all sort of shapes for the term structure.

## 4.6 Explanation of the stylized facts

The five stylized facts can be explained by our model.

With the spot-rate exogenously fixed, sticky inflation and adjustment costs, the Fisher hypothesis of constant real interest rates can't hold and the expected real spot interest rate strays from the expected marginal product

of capital for a while. A positive (negative) inflation shock not accompanied by a spot-rate jump lowers (raises) the real interest rate below (above) the present capital productivity level and sponsors capital investment (disinvestment). But, due to increasing investment costs, capital does not adjust instantaneously.

Inasmuch as the expected inflation is pro-cyclical, (i) the nominal interest rates level is high in the peak and low in the trough of the business cycle.

Pro-cyclical nominal rates means existing bonds are expected to lose (gain) value during the expansion (contraction) as the rates increase (decrease). The negative of the modified duration of the bond, defined as:

$$-M.Duration = \frac{\partial B}{\partial y} \frac{1}{B} = -j \frac{1}{(1+y)}$$

shows that longer bonds are relatively more affected by the expected future change in the level of the term structure. Thus, (ii) the countercyclical pattern of the term spread can be explained as a “level upside-move risk” that is proportional to the bond duration. Due to system stability, people believe there are upper and lower bounds for the expected inflation and the probability of a monetary authority action against inflation is increasing with



inflation itself. When the economy begins an expansion, the nominal interest rates and inflation levels are low, and inflation is expected to grow. Spot rate jumps in the near future will have positive signs, this meaning lower bond prices and capital losses for the long maturities bond holders, who charge their borrower for that. As expansion takes place, inflation increases, followed by the spot-rate. Since there is a perceived upper limit for the inflation, the ‘level upside-move risk’ decreases along this path, and the reduction in the term spread is consistent. The description of the recession goes along the same lines.

Convexity, defined as:

$$Convexity = \frac{\partial^2 B}{\partial y^2} \frac{1}{B} = j(j+1) \frac{1}{(1+y)^2},$$

shows that the (iii) pro-cyclical curvature is explained by the same ‘level upside-move risk’.

The way nominal spot interest rate is modified gives rise to a (iv) negative short-run correlation between expected inflation and expected future real interest rate, since inflation innovations are not instantaneously transmitted to the nominal spot rate.

The monetary authority operating procedure, together with inflation stickiness and the system stability seem enough to justify (v) the better predictability of the slope of the yield curve at the short- and at the long-ends respectively (or the worse predictability of the slope of the middle of the yield curve). The monetary authority operating procedure and inflation stickiness imply the persistency of monetary policy and that inflation lasts for a while, explaining the good predictability of the slope at the short-end of the term structure. At the long-end, because the system is stable, long-term bond yields are mainly defined by the long run values, and shocks have a transitory and small impact. Investors have reasonable certainty about inflation and the spot rate in the near future, as well as in the long run given the system is stable. However, due to the same inflation stickiness and operating procedures, people is uncertain about how long it takes for a policy to reach its goal and when it is going to be reverted, these being the causes for increased middle term uncertainty.

In the context of the present model, we have three shocks that can be decomposed into orthogonal factors, but not interpreted as a factor itself. Our structural shocks are not orthogonal: technology shocks may cause expectational errors and inflation, and inflation may cause spot rate jumps.

Factor 1 for example, which affects all yields with the same sign but affects long yields less, might have considerable weight on the technology,  $\varepsilon$ , and expectational error shock,  $\omega$ , since both impact more short rates and die out with time. We thus let factor interpretation for further research.

## 5 Model Solution, Simulations and Predictions

Equations (4), (13), (25), (26), (21) and (28) form a non-linear stochastic difference system with rational expectations that can be numerically solved according to Novales et al.[21] by use of Sims [26] method described in the Appendix 2.

Numerical exercises reported below used the following set of parameters:  $\alpha = 0.4$  and  $\delta = 0.025$  are standard calibration parameters for quarterly frequency data. Values for  $\sigma = 2$  and  $\beta = 0.995$  are in accordance with Fuhrer's [15] similar model. A  $\varphi = 380$  seems reasonable in view of the existing literature (see Dixit & Pindyck [11]). Finally,  $\rho = 0.9$  and  $\gamma = 0.024$  were estimated from data. The procedure performed to estimate  $\rho$  was close to Cooley&Prescott [9]: first assuming capital does not vary from quarter to

quarter, we have  $\log \theta_t - \log \theta_{t-1} = (\log Y_t - \log Y_{t-1})$ , an expression which allows building up the  $\theta_t$  series, where  $Y$  is the gap between GNP and potential GNP; then, with the obtained  $\theta$ 's,  $\rho$  is estimated. The  $\gamma$  was estimated by instrumental variables using CPI inflation seasonally adjusted and the negative of the System Open Market Accounting Holdings (per-capita and discounted a trend).

## 5.1 Experiments

Figures 8 and 9 illustrate the dynamics of two experiments: (i) a disinflation experiment, when inflation and capital start above the steady state (Figure 8), and (ii) an expansion experiment, when capital as well as inflation start below the steady state level (Figure 9).

As shown in Figure 8, the level of the nominal interest rates are initially high, but the short real interest rate is expected to increase and inflation to decrease. The evolution of the term structure is illustrated in the figure.

In Figure 9, capital and consumption increase along time, while the real interest rates decreases.

In both cases the impulse response functions seem to describe real data<sup>17</sup>.

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<sup>17</sup>More rigorous tests are certainly desirable; comparison with an unrestricted VAR

## 5.2 Simulation with the U.S. data

It is worth asking if the numerical predictions of the theoretical model present patterns similar to the stylized facts in section 2.

In an attempt to test whether the model reproduces the data pattern, we have performed the following Monte Carlo exercise: given date  $t$  states  $\pi_{t-1,t-2}$ ,  $b_t^0$ ,  $k_t$  and  $i_{t+1}$ , to build the joint expectation conditional on the available information set, 500 random paths of the model's variables were obtained by simulating the system 10 years ahead, using shocks got from a bivariate normal random vector with covariance matrix  $\begin{pmatrix} 0.0725 & \\ -.002 & .00579 \end{pmatrix}$ , which is the estimated matrix from the above residual series. With the joint expectation of the model variables calculated, the nominal term structure on  $t$  was then defined by the yields of the many maturity bonds:

$$y_t^j = \frac{1}{\beta} \left( E_t \left[ \frac{1}{1 + \pi_{t+j,t}} \frac{u'(c_{t+j})}{u'(c_t)} \right] \right)^{\frac{-1}{J}} - 1, \quad \forall j = 1, \dots, 40.$$

To move from  $t$  to  $t + 1$ , and calculate the term structure on  $t + 1$  as just described, we assumed the realized shock to be the residual shock  $(\hat{\varepsilon}_t, \hat{\omega}_t)$  estimated from equations (4) and (25) from 1969:1 to 2000:4. The  $\hat{\varepsilon}$  was as

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seeming the natural candidate.

the residual of the equation for estimating  $\rho$ . The  $\widehat{\omega}$  was the residual of the equation for estimating  $\gamma$  (see Section 5 introduction).

The results are sensible and “close” to the qualitative pattern documented in Section 2. Tables 4 and 5 below show the relative importance of the factors and their respective eigenvectors.

The simulation also reproduces the correlation between expected inflation and real interest rate. Table 6 shows the obtained values, which are close to the U.K. empirical ones.

## 6 Conclusion

The simple macro model developed in this paper is able to fit the empirical term structure of interest rates in different situations. It doesn’t focus on the behavior of some instantaneous spot rate process, derived from a particular equilibrium model, to obtain the term structure, as usual in the literature. Instead, it sees the spot-rate as an instrument of the monetary authority, who controls it to match the goal of low price variation. A key behavioral rule introduces the needed flexibility in linking macro variables changes to movements in the yield curve. This being the case, the long run levels of the

state variables may be forecasted with a high degree of accuracy, as well as the future changes in the spot rate. To obtain the term structure, people does take into account the current drift of the inflation and what future monetary policy actions it implies.

Simulations produced results qualitatively close to several stylized facts: (i) pro-cyclical pattern of the level of nominal interest rates; (ii) counter-cyclical pattern of the term spread (as well as low sensitivity of long yields to monetary policy changes); (iii) pro-cyclical pattern of the curvature of the term structure; (iv) lower predictability of the slope of the middle of the yield curve; and (v) negative correlation of changes in real rates and expected inflation at short horizons. Other empirical experiments may show how good is the proposed model to fit various empirical sets of data. From a theoretical viewpoint, new and probably more accurate, bond pricing mechanisms can be developed from it.

## Appendices

### A The Competitive Problem

The equivalence of the representative consumer with a competitive economy is shown below. As usual in the competitive framework, consumers and firms maximize their objective function taking prices as given. Without loss of generality, it is assumed that the firms are the owners of capital and are all equity financed<sup>18</sup>.

#### A.1 Consumers

The consumers budget constraint is given by:

$$\begin{aligned} c_t + q_t z_{t+1} + b_{t+1}^1 + \sum_{j=2}^{\infty} b_{t+1}^j & \quad (A.1) \\ = (q_t + d_t) z_t + w_t l_t + \frac{1}{(1 + \pi_{t,t-1})} \left[ (1 + i_t) b_t^0 + \sum_{j=1}^{\infty} \frac{B_t^j}{B_{t-1}^{j+1}} b_t^j \right] - \tau_t; \end{aligned}$$

and the transversality conditions (6) and:

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<sup>18</sup>For the firms decision between equity and debt in a framework similar as ours, see Brock and Turnovsky (1981). Notice that they deal with such decision in a perfect foresight situation.



$$\lim_{t \rightarrow \infty} \beta^t (q_t + d_t) z_t = 0; \quad (\text{A.2})$$

where:  $q$  is the real stock price;  $z$  is the quantity of the stocks;  $d$  is the real dividends;  $w_t$  denotes the real wage; and  $l_t$  is the amount of labor.

This results in the consumers' optimal allocation rules:

$$1 = \beta E_t \left[ \frac{q_{t+1} + d_{t+1}}{q_t} \frac{u'(c_{t+1})}{u'(c_t)} \right]; \quad (\text{A.3})$$

and (9), (11), taking prices as given.

Given that consumers do not enjoy leisure,  $l_t = 1 \forall t$ .

## A.2 Firms

At the firm's side, the law of motion for its capital stock is:

$$K_{t+1} = K_t^d + I_t^d = (1 - \delta) K_t + I_t^d;$$

where:

$K_{t+1}$  is the capital stock to be used next period;

$K_t^d$  stands for used capital demanded for use next period; and

$I_t^d$  is the real investment on new capital.

Define the gross profits to be given by:

$$Profits_t = f(K_t, l_t, \theta_t) - w_t l_t.$$

Assuming firms are all equity financed, the following identity holds:

$$Profits_t = RE + d_t z_t,$$

and the ex-dividend relation is:

$$q_t z_{t+1} = p_{k, t} K_{t+1};$$

with  $RE$  for retained earnings and  $p_{k, t}$  being the real price for used capital.

Financing of new and used capital obeys:

$$p_{k, t} K_t^d + I_t^d + \varphi \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 = RE + q_t (z_{t+1} - z_t) + (1 - \delta) p_{k, t} K_t;$$

and the net cash flow is defined as:

$$\begin{aligned}
N_t &= f(K_t, l_t, \theta_t) + (1 - \delta) p_{k, t} K_t - w_t l_t - p_{k, t} K_t^d \\
&\quad - \left[ (K_{t+1} - (1 - \delta) K_t) + \varphi \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 \right] \\
&= d_t z_t + q_t (z_t - z_{t+1})
\end{aligned}$$

Thus, the firm problem can be posed as:

$$W(K_t) = \max_{k, l} \{N_t + E_t [M_{1t} W(K_{t+1})]\},$$

with  $M_{1t}$  treated parametrically by firms<sup>19</sup>, and gives the first order conditions:

$$w_t = f(K_t, l_t, \theta_t) - f_1(K_t, l_t, \theta_t) K_t;$$

$$p_{k, t} = E_t [M_{1t} W_1(k_{t+1})]; \quad (\text{A.4})$$

$$\left[ 1 + 2\varphi \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{1}{K_t} \right] = E_t [M_{1t} W_1(K_{t+1})]; \quad (\text{A.5})$$

---

<sup>19</sup>As noted above, in equilibrium, it depends on the consumers' behavior.

The envelope is:

$$W_1(k_t) = f_1(K_t, l_t, \theta_t) - 2\varphi \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{K_{t+1}}{K_t^2} + (1 - \delta) p_{k, t}. \quad (\text{A.6})$$

Substituting the envelope forwarded one period into (A.4) as well as (A.4) into (A.5) results the firms' optimal decision rules:

$$\begin{aligned} p_{k, t} & \quad (\text{A.7}) \\ = E_t \left[ M_{1t} \left\{ f_1(K_{t+1}, l_{t+1}, \theta_{t+1}) - 2\varphi \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}^2} + (1 - \delta) p_{k, t+1} \right\} \right] \end{aligned}$$

and

$$\left[ 1 + 2\varphi \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{1}{K_t} \right] = p_{k, t};$$

taking prices as given.

It is worth pointing that by (A.3) and (32):

$$= \left[ \frac{\left[ \frac{q_{t+1} + d_{t+1}}{q_t} M_{1t} \right]}{\frac{\left\{ f_1(K_{t+1}, l_{t+1}, \theta_{t+1}) - 2\varphi \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}^2} + (1 - \delta) p_{k,t+1} \right\}}{p_{k,t}}} M_{1t}} \right];$$

and  $p_{k,t}$  is equal to Tobin's marginal  $Q$  is given by:

$$p_{k,t} = 1 + 2\varphi \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{1}{K_t} = Q;$$

### A.3 Competitive Economy Equilibrium

An equilibrium is defined as a set of stochastic processes

$(r_{t,t+l}^j, q_t, p_{k,t}, z_t, b_{t+1}^j, k_{t+1}, c_t, l_t)$  satisfying the f.o.c.'s and the market clearing conditions.

Because  $l_t = 1 \forall t$ , we can argue in terms of per capita capital  $k_t$ .

To make things simpler, assume there is no issue of new shares and the firm finances itself by retained earnings:

$$z_{t+1} = z_t = 1,$$

which implies, by the ex-dividend relation and the market clearing that:

$$q_t = p_{k,t} k_{t+1} \quad \forall t.$$

The economy resources constraint is thus:

$$f(k_t, \theta_t) = c_t + \left[ (k_{t+1} - (1 - \delta) k_t) + \varphi \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right], \quad (\text{A.8})$$

and given the model parameters, the economy equilibrium conditions become:

$$p_{k,t} = \beta E_t \left[ \left\{ f_1(k_{t+1}, 1, \theta_{t+1}) - 2\varphi \left( \frac{k_{t+1}}{k_t} - 1 \right) \frac{k_{t+1}}{k_t^2} + (1 - \delta) p_{k,t+1} \right\} \frac{u'(c_{t+1})}{u'(c_t)} \right]$$

and

$$\left[ 1 + 2\varphi \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{1}{K_t} \right] = p_{k,t},$$

with  $c_t$  given by (A.8). A system of two simultaneous equations that can be solved for the two unknowns  $p_k$  and  $k$  (or  $Q$  and  $k$ ).

## B Numerical Solution

The non-linear stochastic difference system with rational expectations

(4), (13), (25), (26), (21) and (28) can be linearized around the steady and solved by some linear solution methods with reasonable precision, as shown in Novales et al. [21].

We have chosen to use Sim's [26] method to solve our model. The procedure consists of dealing with each conditional expectation and the associated expectational error as additional variables, adding to the system an equation that defines the expectational error. In our case, we have defined the variables:

$$W_{1t} = E_t \left\{ \left[ \alpha \theta_{t+1} k_{t+1}^{\alpha-1} + (1 - \delta) - 2\varphi \left( \frac{k_{t+2}}{k_{t+1}} - 1 \right) \frac{k_{t+2}}{k_{t+1}^2} \right] u'(c_{t+1}) \right\},$$

$$W_{2t} = E_t \left[ \frac{1}{(1 + \pi_{t+1,t})} u'(c_{t+1}) \right],$$

$$W_{3t} = E_t [\pi_{t+1,t}];$$

and the respective expectational errors  $\eta_{1t}$ ,  $\eta_{2t}$ ,  $\eta_{3t}$ .

The resulting linearized system is then written as:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t$$

where:

$y_t = (c_t - c_{ss}, k_{t+1} - k_{ss}, b_{t+1} - b_{ss}, W_{1t} - W_{1ss}, W_{2t} - W_{2ss}, i_{t+1} - i_{ss}, \log \theta, \pi_{t,t-1}, W_{3t}, b_t - b_{ss})$  is the vector of variables determined within the model (inclusive  $W$ , but except  $\eta$ ) and  $\Gamma_0$  and  $\Gamma_1$  are the matrices containing the system linearized coefficients.

If  $\Gamma_0$  is invertible (and it is):

$$y_t = \Gamma_0^{-1}\Gamma_1 y_{t-1} + \Gamma_0^{-1}\Psi z_t + \Gamma_0^{-1}\Pi \eta_t = \widetilde{\Gamma}_1 y_{t-1} + \widetilde{\Psi} z_t + \widetilde{\Pi} \eta_t$$

and  $\widetilde{\Gamma}_1$  has a Jordan decomposition  $\widetilde{\Gamma}_1 = P\Lambda P^{-1}$ .

Defining  $w_t = P^{-1}y_t$ , we obtain:

$$w_t = \Lambda w_{t-1} + P^{-1} \left( \widetilde{\Psi} z_t + \widetilde{\Pi} \eta_t \right)$$

where, for every eigenvalue  $\lambda_j$  of  $\widetilde{\Gamma}_1$  we have an equation:

$$w_{jt} = \lambda_{jj} \Lambda w_{j,t-1} + P^{j\cdot} \left( \widetilde{\Psi} z_t + \widetilde{\Pi} \eta_t \right)$$

where  $P^{j\cdot}$  denotes the  $j$ -th row of  $P^{-1}$ .

As the state variables and the shadow prices are assumed to grow less



than  $\beta^{-\frac{1}{2}}$ , for  $w_j$ , with  $|\lambda_{jj}| > \beta^{-\frac{1}{2}}$ , we need have:

$$w_{jt} = P^{j\bullet} y_t = 0, \quad \forall t$$

which provides the system stability conditions.

Using the notation  $P = [P_{\bullet S}, P_{\bullet U}]$  and  $P^{-1} = \begin{bmatrix} P^{S\bullet} \\ P^{U\bullet} \end{bmatrix}$ , where U stands

for “unstable”, the system can be written as:

$$w_{S,t} = \Lambda_S w_{S,t-1} + \begin{bmatrix} I & -\Phi \end{bmatrix} P^{-1} \Psi z_t; \quad (\text{B.1})$$

where:  $\Phi = P^{S\bullet} \Pi \eta (P^{U\bullet} \Pi \eta)^{-1}$ .

To arrive at an equation in  $y$ , we use  $y = Pw$  to transform (B.1) into:

$$y_t = P_{\bullet S} \Lambda_S P^{S\bullet} y_{t-1} + (P_{\bullet S} P^{S\bullet} - P_{\bullet S} \Phi P^{U\bullet}) \Psi z_t,$$

which can be solved after imposing:

$$w_{U,0} = P^{U\bullet} y_0 = 0.$$

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## Figures and Tables

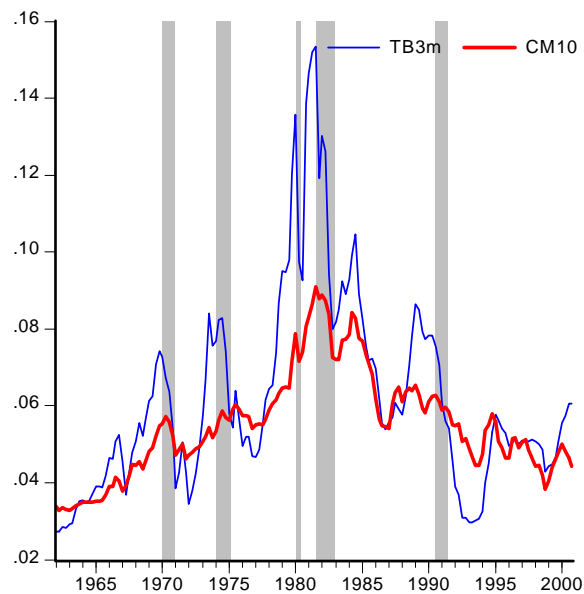


Figure 1: Evolution of U.S. nominal yields from 1962:01 to 2000:04

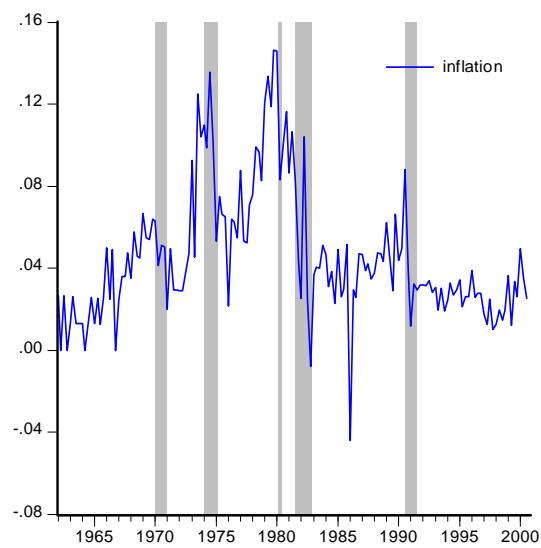


Figure 2: U.S. quarterly inflation from 1962:1 to 2000:4

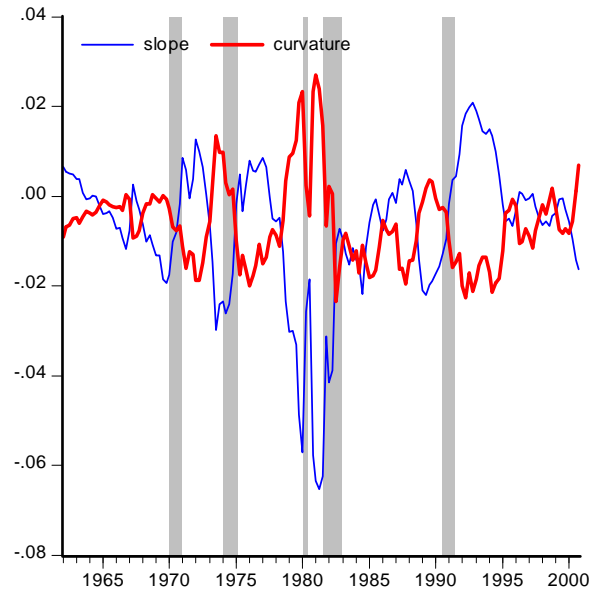


Figure 3: Evolution of U.S. slope and curvature of the term spread from  
1962:03 to 2000:04

	2	3	6	12	24	48	120
b	.502	.467	.320	.272	.363	.442	1.402
(s.e.)	(.096)	(.148)	(.146)	(.208)	(.223)	(.384)	(.142)

Table 1: b estimates by Campbell, Loo and MacKinlay



Maturity	1st. P.C.	2nd. P. C.	3rd. P. C.
TB3m	0.516	-0.525	0.598
TB6m	0.521	-0.235	-0.251
TB1	0.502	0.054	-0.618
CM3	0.317	0.433	-0.004
CM5	0.237	0.447	0.177
CM7	0.185	0.404	0.259
CM10	0.140	0.340	0.314

Table 2: Empirical First three principal components (or eigenvectors with largest eigenvalues) - quarterly U.S. nominal yields from 1969:03 to 2000:04

Maturity	Total Variance Explained by	Proportion of Total Explained Variance Accounted for by		
	Factor1+Factor2	Factor 1	Factor 2	Factor 3
TB3m	99.1	92.0	7.1	0.8
TB6m	99.6	98.1	1.5	0.2
TB1	98.9	98.8	0.1	1.0
CM3	99.6	87.4	12.2	0.0
CM5	99.4	78.5	20.9	0.3
CM7	98.7	72.8	25.9	1.0
CM10	95.8	66.6	29.3	2.3
<b>Average</b>	<b>98.7</b>	<b>84.9</b>	<b>13.8</b>	<b>0.8</b>

Table 3: Empirical Relative Importance of the Empirical Factors (%) - quarterly U.S. nominal yields from 1969:03 to 2000:04

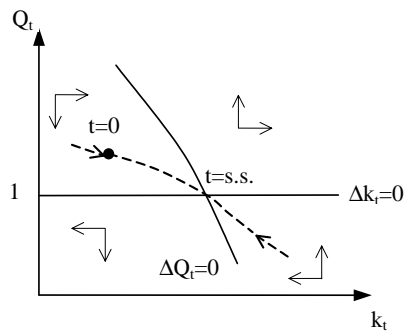


Figure 4: Q x k phase diagram

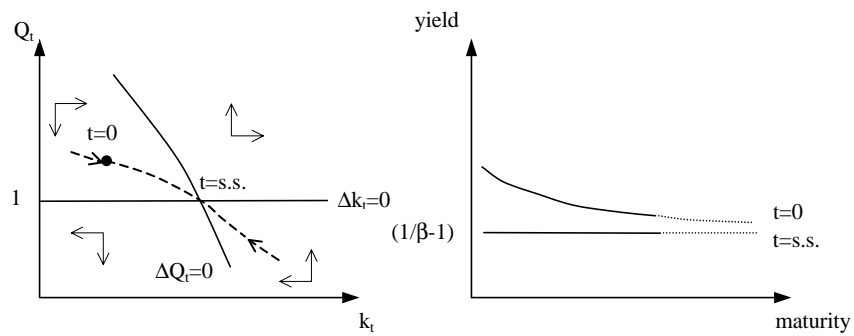


Figure 5: Expansion path and the real term structure without inflation

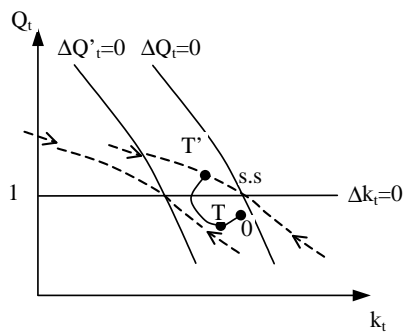


Figure 6: Certain transitory tight

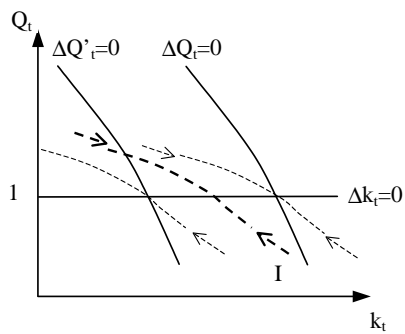


Figure 7: Uncertain transitory tight

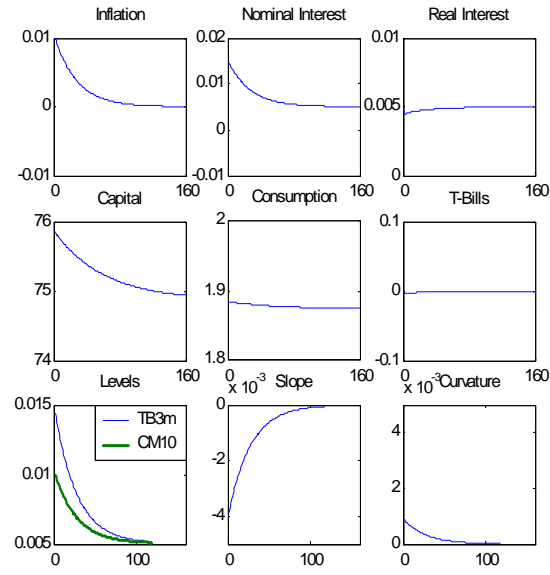


Figure 8: Contraction path

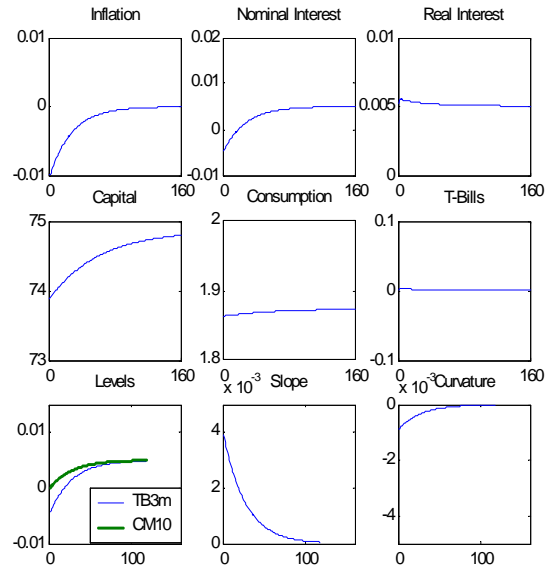


Figure 9: Expansion path

Maturity	1st. P.C.		2nd. P. C.		3rd. P. C.	
	Simul.	Empir.	Simul.	Empir.	Simul.	Empir.
TB3m	<b>0.600</b>	0.516	<b>-0.678</b>	-0.525	<b>0.413</b>	0.598
TB6m	<b>0.499</b>	0.521	<b>-0.065</b>	-0.235	<b>-0.730</b>	-0.251
TB1	<b>0.424</b>	0.502	<b>0.337</b>	0.054	<b>-0.243</b>	-0.618
CM3	<b>0.291</b>	0.317	<b>0.371</b>	0.433	<b>0.245</b>	-0.004
CM5	<b>0.241</b>	0.237	<b>0.340</b>	0.447	<b>0.242</b>	0.177
CM7	<b>0.204</b>	0.185	<b>0.299</b>	0.404	<b>0.269</b>	0.259
CM10	<b>0.162</b>	0.140	<b>0.282</b>	0.340	<b>0.217</b>	0.314

Table 4: Simulated and Empirical First three principal components (or eigenvectors with largest eigenvalues) - quarterly U.S. nominal yields from 1969:03 to 2000:04

Maturity	Total Variance Explained by		Proportion of Total Explained Variance Accounted for by					
	Factor1+Factor2		Factor 1		Factor 2		Factor 3	
	Simul.	Empir.	Simul.	Empir.	Simul.	Empir.	Simul.	Empir.
TB3m	<b>97.3</b>	99.1	<b>76.8</b>	92.0	<b>20.5</b>	7.1	<b>2.7</b>	0.8
TB6m	<b>84.8</b>	99.6	<b>84.5</b>	98.1	<b>0.3</b>	1.5	<b>13.4</b>	0.2
TB1	<b>91.0</b>	98.9	<b>80.4</b>	98.8	<b>10.6</b>	0.1	<b>2.0</b>	1.0
CM3	<b>91.4</b>	99.6	<b>68.3</b>	87.4	<b>23.1</b>	12.2	<b>3.6</b>	0.0
CM5	<b>92.0</b>	99.4	<b>64.9</b>	78.5	<b>27.0</b>	20.9	<b>4.9</b>	0.3
CM7	<b>87.0</b>	98.7	<b>60.1</b>	72.8	<b>26.9</b>	25.9	<b>7.7</b>	1.0
CM10	<b>84.2</b>	95.8	<b>51.7</b>	66.6	<b>32.5</b>	29.3	<b>6.8</b>	2.3
<b>Average</b>	<b>89.7</b>	98.7	<b>69.5</b>	84.9	<b>20.1</b>	13.8	<b>5.9</b>	0.8

Table 5: Simulated and Empirical Relative Importance of the Empirical Factors (%) - quarterly U.S. nominal yields from 1969:03 to 2000:04

Maturity	Correlations	
	Simul.	Empir.
3-month	-0.87	
6-month	-0.60	
1-year	-0.40	-0.69
3-year	-0.35	
5-year	-0.30	-0.06
7-year	-0.22	
10-year	-0.06	-0.08

Table 6: Simulated correlations between the same maturities real rate and  
expected interest rate